JOURNAL OF FORENSIC SCIENCES

J Forensic Sci, November 2009, Vol. 54, No. 6
doi: 10.1111/j.1556-4029.2009.01151.x
Available online at: interscience.wiley.com

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Upper-Bounding the Incidence Rate of Associations Between Camouflage Uniforms and Surveillance Images*

ABSTRACT: Camouflage garments can be associated with surveillance images of a crime scene even in the absence of unique wear marks or very high-quality images. However, the probability of an accidental association, or incidence rate, is significant. The present work describes and validates a method for estimating the incidence rate based on a statistical model of the garment manufacturing process. The model was developed primarily for use with the current U.S. Army Combat Uniform (ACU), but can be applied to any camouflage garment. Eight garment manufacturers were studied, and all sources of variation in the manufacturing process were characterized. The marking and spreading procedures were found to be dominant and consistent sources of variation. However, some sources of variation, in particular those because of human operators, were not consistent enough to accurately characterize. Sources of variation that could not be well-characterized were ignored in the statistical model, yielding a worst-case estimate that is an upper-bound to the true incidence rate. The model was evaluated for a variety of cases. Depending on the quality of the surveillance image, the manufacturing parameters, and the local population, incidence rates range from about 3% to negligibly small. The model was validated by returning to one manufacturer, and sampling a large number of completed garments and estimating empirical match probabilities. The empirical probabilities validated the estimates of the worst-case incidence rate and also demonstrated that typical incidence rates are significantly lower.

KEYWORDS: forensic science, individualization, statistical modeling, incidence rate, camouflage garments, surveillance images

Forensic examiners are frequently presented with evidence collected at a crime scene that can be associated with evidence seized from a suspect. Such associative evidence may be presented in court to support the prosecution if it can be shown that the probability of an accidental association is very small. The probability of an accidental association is the *incidence rate* of that particular association.

Often it is impossible to know, or estimate, the true incidence rate. This is because, generally, not enough is known about the match characteristics or subject population to statistically model the association. In lieu of the incidence rate, it is common practice for forensic examiners to only present an association if sufficient match characteristics are identified so as to preclude the *possibility* of an accidental association (1). However, several recent studies demonstrate that it is never the case that the probability of accidental association is strictly zero, even with a large number of precise match characteristics (2–5). This has motivated recent research in estimating incidence rates for forensic investigations (6–9).

In the present work, we describe a statistical model for computing the incidence rate for a specific class of associations: associations between a camouflage garment depicted in surveillance images and a physical garment seized from a suspect. Garments made from camouflage prints exhibit consistent statistical properties that make incidence rate estimation feasible. However, as incidence rate estimation is necessarily an estimation problem, we develop a statistical model for an upper-bound on the incidence rate (i.e., a worst-case incidence rate). Such an upper-bound on the incidence rate would allow forensic examiners and prosecutors to determine the strength of the association. This statistical model has been developed for United States Army digital camouflage uniforms (the Army Combat Uniform or ACU) for forensic investigations by the U.S Army Criminal Investigation Laboratory (10). The model is general, however, and can be applied to any investigation that involves surveillance images of camouflage garments in which the underlying camouflage pattern and garment manufacturing parameters are known.

The remainder of this section provides background information on garment associations and incidence rate statistics. Statistical Characterization describes the camouflage garment manufacturing processing and characterizes all relevant sources of variation. Probability Calculation describes the statistical model for calculating the incidence rate. Model Analysis describes an example of the model and highlights important trends in the incidence rate for typical parameters. Finally, Model Validation reports on an empirical study that validates the statistical model. The research and model described in this paper have been incorporated into a forensic software tool, the Military Uniform Uniqueness Statistical Evaluator (MUUSE) (10). For additional details on MUUSE, please contact Quantum Signal LLC at www.quantumsignal.com.

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*This project was supported by the Technical Support Working Group (TSWG).

Received 8 April 2008; and in revised form 24 June 2008, 13 Oct. 2008; accepted 13 Nov. 2008.

Garment Associations

Consider a criminal investigation in which a surveillance image from a crime scene depicts the perpetrator wearing an unspecified garment, and a similar looking garment is seized from a suspect (their home, locker, etc.). The physical and imaged garments can be *individualized* only if specific unique characteristics can be identified in both. For example, individualization may be possible if the suspect has introduced unique characteristics on the garment (wear marks, etc.), or the surveillance image is of such high quality that fine minutiae in the garment are observed (stitching patterns, etc.) (11). However, most "new" garments, when viewed from a distance or through typical surveillance cameras, are indistinguishable from all other new garments of the same type and size. As such, associations between typical "new" garments and surveillance images usually carry little or no evidentiary value.

In contrast, associations involving garments manufactured from camouflage prints may carry significant evidentiary value even in the absence of specific unique characteristics. This is because few camouflage garments are likely to have the same portion of the camouflage pattern at the same location on the garment. However, while most garments are differentiable, duplicates are possible owing to the finite, repeating camouflage pattern. Furthermore, the probability that two camouflage garments are indistinguishable increases considerably if either garment is viewed with low quality surveillance images.

For criminal investigations involving military personnel, the widespread use of standard issue camouflage uniforms such as the ACU presents a unique context in which *statistical* garment association is feasible. In this case a great deal is known about both the population of available garments. In particular, the garments are well specified (12) and the manufacturing process is highly standardized (13). Hence a statistical model can be built that accounts for all significant sources of variation in the garment.

A previous study attempted to utilize this observation, but did not build a statistical model based on the manufacturing process (9). Rather, the study took an empirical approach to the problem, sampling a few hundred garments and computing the number of matches. As such, there is a question as to whether the results of this study can be generalized beyond the relatively small population from which the garments were sampled. In the work described in this document, the ability to study and model the manufacturing process enables a first-principles approach that can reliably be generalized. The authors have visited numerous ACU manufacturing facilities and quantified all significant sources of garment variation. With this knowledge a statistical model was built that, when coupled with parameters estimated from crime scene evidence, computes an upper-bound to the incidence rate.

Likelihood Ratio and the Principle of Individualization

The process of individualizing a garment portrayed in surveillance images taken at a crime scene is the same as that behind the comparison of fingerprints, footprints, and many other types of physical evidence (11). As described by Tuthill and George (14)

> The individualization of an impression is established by finding agreement of corresponding individual characteristics of such number and significance so as to preclude the possibility (or probability) of their having occurred by mere coincidence, and establishing that there are no differences that cannot be accounted for.

In this study, the *impression* is the garment portrayed in the surveillance images, and this impression is compared to a physical garment seized from a suspect. Traditionally, forensic examiners only associate impressions with the suspect that preclude the *possibility* of an accidental association. But what if only enough characteristics are observable to preclude the *probability* of an accidental association? In this case, a forensic examiner is often obliged to abandon the association, as any uncertainty in the prosecution's case may be grounds for acquitting the defendant. On the other hand, if the uncertainty can be quantified, then the association may be presented in court. The incidence rate is one such measure of the uncertainty, or strength, of an association (15,16).

The incidence rate is related to another commonly used measure of association strength, the likelihood ratio (6). Let E_C be the event that a surveillance image from the crime scene that depicts a camouflage garment *matches* a camouflage garment seized from a suspect. By "matches" we mean that a forensic examiner has established that the two pieces of evidence are compatible: there is agreement between all observable characteristics, and there are no differences that are unaccounted for. The likelihood ratio for this match is

$$\mathrm{LR} = \frac{\Pr(E_c|H_1)}{\Pr(E_c|H_0)}, \tag{1}$$

where LR is the likelihood ratio, H_0 and H_1 are the two hypotheses—H_1: the garment seized from the suspect is the garment depicted in the surveillance image and H_0: the garment seized from the suspect is *not* the garment depicted in the surveillance image.

Standards have been proposed for weighting evidence E_C based upon the likelihood ratio. For example, the British Forensic Science Service employs a standard in which ratios between 10 and 100 provide "moderate support" to the prosecution's case (7).

H_0 is the hypothesis that the seized and imaged garments are two distinct garments. Hence $\Pr(E_c|H_0)$ is the probability, or incidence rate, of a match occurring between the surveillance image and a randomly chosen garment. This component of the likelihood ratio is the focus of this study. In many instances, it may be reasonable to assume that $\Pr(E_c|H_1) \approx 1$, in which case the likelihood ratio is approximately the reciprocal of the incidence rate. The validity of this assumption depends upon the expert opinion of the forensic examiner who matched the seized garment to the surveillance image. If the forensic examiner concludes with certainty that had the seized garment been worn by the perpetrator it would have been indistinguishable from the surveillance image, then this implies $\Pr(E_c|H_1) \approx 1$.

It is not always reasonable to assume $\Pr(E_c|H_1) \approx 1$, however. For example, when viewed from a surveillance camera, the wrinkles and folds that are present in most camouflage garments distort the apparent pattern. Often it is difficult to determine the shape of the wrinkles and folds from a surveillance image, and hence the forensic examiner may not be able to determine if a discrepancy between the seized garment and the surveillance image is due to wrinkles and folds, or due to a difference in the pattern. In this case, $\Pr(E_c|H_1) < 1$. In this study, we consider the scenario in which the forensic examiner concludes with certainty that the seized garment is indistinguishable from the surveillance image. The remainder of this study considers the incidence rate alone.

Statistical Characterization

To upper-bound the incidence rate of ACU garment matches, we propose a framework for piece by piece matching of ACU garments. Each ACU garment is constructed from a fixed number of pieces of fabric. For example, ACU coats are constructed from 26 pieces and trousers from 39 pieces. The camouflage pattern is the same for all fabric used in ACU garments, and repeats periodically with a period L of ~34.2 inches in both directions. Manufacturers of ACU garments are required to orient the fabric in one of two possible directions for all piece in the garment, but the portion of camouflage pattern that is visible in each piece is not otherwise specified. Accordingly, two pieces of fabric with the same shape and orientation are "identical" if a reference point in both pieces is located at the same position in the L-by-L camouflage pattern.

However, in the present work, we are concerned with forensic applications in which only one garment is physically available—the garment seized from a suspect. The other garment is depicted in a surveillance image. Surveillance images are often low-resolution, taken under poor lighting, contain compression artifacts, and depict garments that are wrapped around complex surfaces (a human body). In this case, the reference points from the same pieces of the two garments do not need to come from exactly the same location in the camouflage pattern for the two pieces to be judged as matching. There always exists a range of shifts for each reference point within which the mismatch cannot be detected. We define the size of this range as the *uncertainty* of the match. The uncertainty depends on the surveillance image, may be different for each observable piece in the image, and must be determined by the forensic examiner.

In addition to the uncertainty range of the match, two unique pieces can be erroneously judged as identical if they are drawn from distinct locations of the camouflage pattern in which the pattern is similar (e.g., see Fig. 1*b* and *c*). Because of the algorithmic method used to generate the ACU pattern, the pattern contains several areas that are very similar, and are indistinguishable in low-quality images. We define such similar portions of the pattern as *siblings*. Some portions of the pattern have many siblings, whereas others are clearly differentiable. For a given match between a piece of the seized garment and a piece depicted in a surveillance image, the number of siblings in the camouflage pattern that would yield the same image is defined as the *multiplicity* of the match. By definition, the match multiplicity is at least one, and every portion of the camouflage pattern has at least one sibling (the portion itself). The match multiplicity depends on the size of observed portion of the pattern, and the image quality: the smaller the portion, the higher the multiplicity, and the poorer the image quality, the higher the multiplicity.

Mathematically, a match between two pieces, *A* and *B*, can be expressed as

$$|x_A - x_B - s_x| < \Delta, \quad |y_A - y_B - s_y| < \Delta, \qquad (2)$$

where (x_A, y_A) are the coordinates of the reference point of piece *A* in the camouflage pattern, (x_B, y_B) are the coordinates of the reference point in piece *B*, (s_x, s_y) are the shifts between the true match location and a sibling, and Δ is the match uncertainty. Note that $(s_x, s_y) = (0,0)$ for the match at the true match location. The coordinates (x_A, y_A) and (x_B, y_B) are measured within a single period of the camouflage pattern, and the difference operator above operates on a circular space $[0, L)$.

A statistical model for the variables in Eq. (2) is developed below. This model is based on the garment manufacturing process, which is briefly described below. The model depends upon the specific manufacturing parameters for each garment, and in particular if the two garments were manufactured from the same *marker,* and if *ply integrity* was maintained for both.

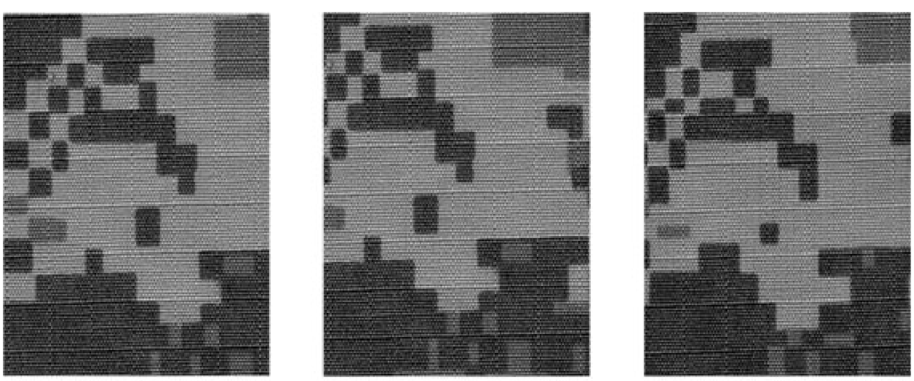
Manufacturing Process

The ACU garment manufacturing process is consistent with methods used throughout the garment industry (13). The process of constructing garments from the camouflage fabric may be broadly divided into four major steps: marking, spreading, cutting, and sewing.

Marking is the process of creating a template for cutting all of the pieces needed for the garment from the camouflage fabric. This template is called a *marker*. Markers usually occupy the full width of the fabric roll minus a small margin on either side. Two example markers are shown below in Fig. 6. Markers are designed using either manual layout software, automatic layout software, or a combination of both. The principal goal in marker design is to minimize the quantity of scrap fabric, without complicating the cutting process unduly. A marker may be used only once or many times over a period of years. The ACU garments are made from a "ripstop" material, and the garment specifications restrict the possible rotations of the pieces in the marker to either one specific orientation, or a 180° rotation of that orientation.

Spreading is the process of laying plies of fabric on top of each other. A stack of plies is termed a *spread*. Different manufacturers use different spread thicknesses, from a few dozen to a few hundred plies, as well as different spread lengths, from several to a hundred yards. Typically, several hundred garments are produced from a single spread. Spreading is accomplished either manually or through automated spreading machines. Two methods of spreading are common: single-sided and double-sided. Single-sided spreads consist of plies that all face one direction, whereas double-sided spreads consist of pairs of plies that face each other. When spreading is complete, a marker is placed on top of the spread, and cutting begins.

Cutting can be performed either manually (a skilled laborer guiding a saw through the plies) or by an automated, computerized



(a) (b) (c)

FIG. 1—*Two types of matches: the same portion of pattern (a–b) and two distinct portions (b–c), both with a small shift within uncertainty range. The portion in (c) is referred to as a "sibling" of (a).*

cutting machine. In either case, the marker is used as a guide for cutting out stacks of pieces. The garment specifications generally dictate that cuts must be within a small margin of error, typically 1/8″.

The stacks of pieces are handled with care as they move through the sewing process. In an effort to maintain color shading consistency across the garment, specifications often require *ply integrity*, so as to ensure that all the pieces used to make a given garment come from the same ply in the spread. Most ACU garment manufacturers enforce strict ply integrity for all medium and large pieces, but many do not maintain ply integrity for small pieces, and some make little effort to maintain ply integrity as they see little variation in the color from ply to ply.

The stacks then move through a factory-style assembly line of sewing stations. Typically, each station is operated by a single individual, and performs a discrete sewing step. Some sewing stations consist of a conventional sewing machine and operator, whereas others consist of an elaborate computer-controlled device that performs one or more alignments, stitches, and/or folds. Manufacturing facilities have varying levels of automation, with some being almost entirely manual and others being almost entirely automatic.

Quality control is enforced by all manufacturers. At the end of the assembly line, garments are inspected for stitch quality, color consistency, and other factors. Defective garments are either repaired or discarded.

Coordinate Definitions

Piece-wise garment matches are determined, mathematically, by the reference coordinates with respect to the camouflage pattern. Therefore, a statistical characterization of the coordinates is required to estimate the incidence rate. Let (x_i,y_i) be the longitudinal (along the spread) and transverse (across the spread) coordinates of the position in the camouflage pattern of a reference point for piece i. These coordinates can be expressed as

$$\begin{aligned} x_i &= x_M + x_{M,i} + x_{cs,i} \\ y_i &= y_M + y_{M,i} + y_{cs,i} \end{aligned} \tag{3}$$

where (x_M,y_M) are the coordinates of the marker with respect to the camouflage pattern, (x_{Mi},y_{Mi}) are the coordinates of the i^{th} piece within the marker, and $(x_{cs,i},\ y_{cs,i})$ are the combined cutting and stitching errors. The cutting and stitching errors are defined as the deviations of the reference position from the nominal because of imperfections in the cutting and stitching processes. These errors are explicitly limited to a small range by the garment specifications, for example less than 1/8″.

Single-Sided Statistics

Each ply of a spread can begin at any point in the L = 34.2″ pattern period, hence the longitudinal coordinate of the marker, x_M, is unrestricted and can assume any value in the [0,L) interval with uniform probability. In contrast, the transverse coordinate, y_M, ideally should be a constant for all plies in the spread. However, in practice there are two factors that cause variation of y_M within a narrow range. First, the camouflage pattern tends to "drift" transversely, i.e., the fabric edge corresponds to different points of the pattern at different locations along the length of the roll (Fig. 2*a* below). Second, the fabric also tends to "drift" transversely as it is spread, i.e., the edges of the spread fabric are not straight and do not align perfectly from ply to ply (Fig. 2*b* below). The former factor is determined by the fabric manufacturer and is typically less than a quarter inch. The latter factor is determined by the spreading process, and its magnitude varies significantly between ACU manufacturers.

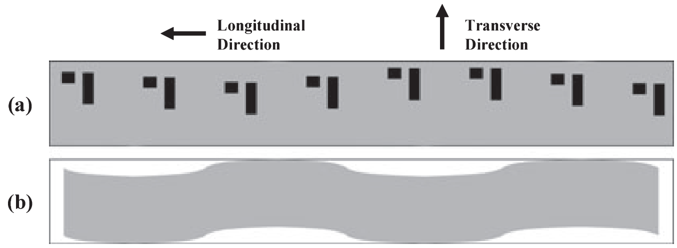


FIG. 2—*Transverse variation of y_M: (a) pattern "floating" within fabric and (b) fabric "floating" along the spread.*

Based on observations of the spreading process, and because the transverse shift is the sum of multiple hidden random variables, it is reasonable to approximate the probability distribution function (PDF) as normal in shape. However, the PDF for the transverse distribution is not strictly normal, as the normal PDF is unbounded and the transverse shift is bounded. In the development below, it is not necessary to assume that the longitudinal distribution is uniform and the transverse distribution is normal. Nonparametric distributions derived directly from the sampled data can be used. However, this would complicate the mathematical development below and does change the ultimate incidence rates appreciably. For simplicity, we approximate the longitudinal distribution as uniform and the transverse distribution as normal.

To estimate the standard deviation of the transverse shift, as well as to verify that the longitudinal shift distribution is approximately uniform and the transverse shift distribution is approximately normal, we developed and deployed a method of direct sampling of the spreads for each manufacturer. The method is based on collecting and analyzing fabric pieces cut from the spread during the manufacturing process. Every marker contains unused areas that produce scrap stacks (Fig. 3*a*), which are typically discarded by the manufacturer. The authors have collected numerous full scrap stacks from eight manufacturers. Between ~100 and 1000 pieces total were collected from each manufacturer. All pieces in each scrap stack are carefully scanned, automatically aligned (Fig. 3*b*), and located in the camouflage pattern (Fig. 3*c*) using custom software based on the cross-correlation surface.

For all pieces in one scrap stack, the second terms in Eq. (3) are constant and the third terms are zero. The variations in (x_i,y_i) are due solely to the shifts in the marker coordinates (x_M,y_M) between plies. Empirical distributions of x_M and y_M are constructed using the aligned reference points for each manufacturer, after removing the mean and discarding outliers from the reference points for each stack.

Sample distributions for one manufacturer are shown in Fig. 4. The histograms show the distribution of the reference points, and the curve in the right panel shows the best fit normal PDF. As expected from general considerations, the longitudinal distribution is approximately uniform, and the transverse distribution is approximately normal. For all manufacturers, the estimated reference points satisfied a Kolmogorov–Smirnov test (with significance level α = 0.05) for a uniform distribution in the longitudinal direction and a normal distribution in the transverse direction. The standard deviation of the transverse distribution was found to vary significantly between manufacturers (~0.15–0.4″).

Double-Sided Statistics

In addition to the marker coordinate distributions, the dependence of the coordinates between plies is important for garment

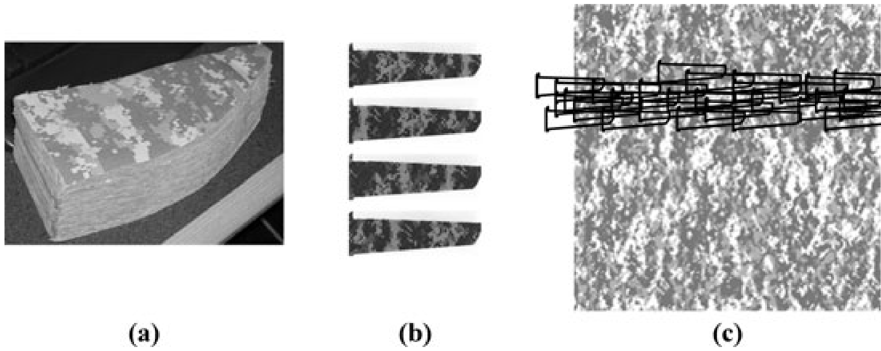


FIG. 3—*Stages of data collection: (a) stacks of scrap pieces cut in the manufacturing process; (b) scanned pieces from one stack; and (c) scrap pieces aligned with the camouflage pattern, allowing one to estimate their coordinate distributions.*

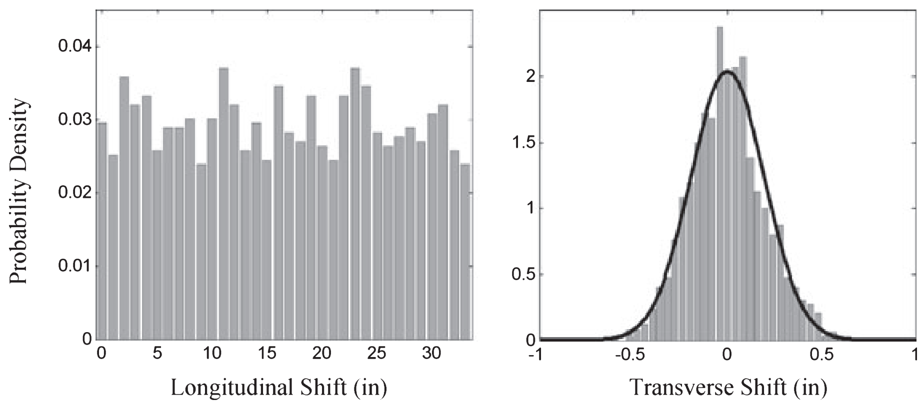


FIG. 4—*Typical empirical histograms of the marker coordinates x_M (a) and y_M (b). The dark line in the right panel shows a Gaussian approximation to the transverse distribution with $\sigma_T = 0.2''$. These empirical distributions were generated from 1587 scanned pieces from one manufacturing facility.*

matching. We found that, in general, plies are independent—that is, knowing (x_M, y_M) for a certain ply does not provide any information regarding another (randomly chosen) ply. However, this is not the case for adjacent plies, which is relevant for double-sided spreading, where the pieces of the same garment often come from adjacent plies. Dependence between adjacent plies is described by the conditional PDFs, $Pr(x_{M,k}|x_{M,k-1})$ and $Pr(y_{M,k}|y_{M,k-1})$. The number of available sample points is insufficient to directly estimate the necessary parameters for every value of $x_{M,k-1}$ and $y_{M,k-1}$. For the longitudinal direction, the conditional distribution is equivalent (same shape, different mean) to the distribution of the difference $(x_{M,k} - x_{M,k-1})$, which is independent of $x_{M,k-1}$.

$$\Pr(x_{M,k}|x_{M,k-1}) \sim \Pr(x_{M,k} - x_{M,k-1}|x_{M,k-1}) = \Pr(x_{M,k} - x_{M,k-1}) \quad (4)$$

The PDF on the right side of Eq. (4) can be estimated from the scrap stacks. Figure 5*a* shows a sample distribution. If the fabric in the spread is laid continuously from the same roll, the distribution acquires an approximately normal shape, as the length of each ply is approximately constant. If there is a splice point, because of a change of fabric rolls, or any other irregularity, then the coordinates $x_{M,k}$ and $x_{M,k-1}$ become decoupled, which corresponds to the uniform distribution tails in Fig. 5*a*. However, the frequency of these irregularities is dependent upon varying manufacturer practices and is difficult to characterize. To maintain the upper-bound described

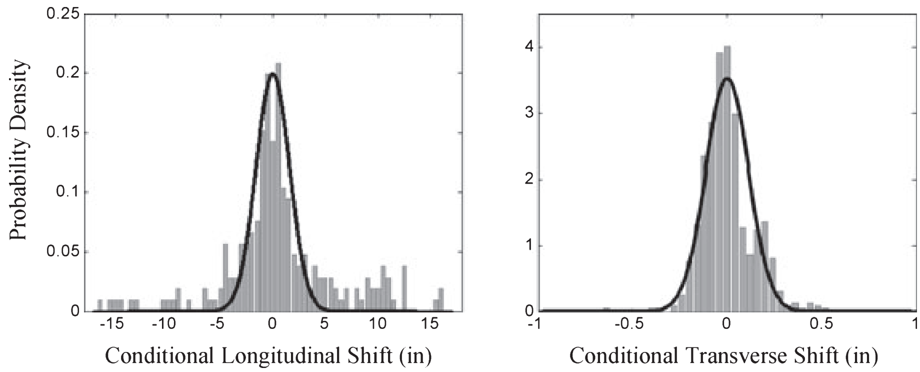


FIG. 5—*Double-sided distributions: zero-mean empirical conditional distribution of the longitudinal (a) and transverse (b) coordinates. The dark curves show the fitted Gaussian model PDFs. These empirical distributions were generated from 384 scanned pieces from one manufacturing facility.*

below, we assume that there are no such irregularities in the spread, which narrows the distribution and increases the probability of an accidental match.

$$\Pr\left(x_{M,k}\middle|x_{M,k-1}\right) \sim N\left(\mu, \sigma_{CL}^2\right), \quad \mu = x_{M,k-1} + const \qquad (5)$$

Transverse marker coordinates in adjacent plies $y_{M,k}$ and $y_{M,k-1}$ are also highly dependent, with correlation coefficients r varying from 0.2 to 0.8 for different manufacturers. Assuming that they conform to a joint normal distribution, we obtain the conditional PDF $Pr(y_{M,k}|y_{M,k-1})$, which is also normal, with non-zero mean and reduced variance (17).

$$\Pr\left(y_{M,k}\middle|y_{M,k-1}\right) \sim N\left(\mu, \sigma_{CT}^2\right), \ \mu = y_{M,k-1}r, \ \sigma_{CT}^2 = \sigma_T^2\left(1-r^2\right) \quad (6)$$

Figure 5*b* shows a sample conditional distribution constructed from a scrap stack by subtracting the expected value $y_{M,k-1}r$ from $y_{M,k}$ for every ply.

Individual-Piece Statistics

Except for the term (x_M,y_M), all of the terms in Eq. (3) are unique for each piece of the garment. In contrast, the term (x_M,y_M) is often, but not always, constant for all pieces in the garment. In fact (x_M,y_M) is the same for all pieces in a garment if and only if ply integrity is maintained for that garment. If two pieces come from different plies, then each piece will have a different (x_M,y_M) term. Hence knowledge of whether or not ply integrity was maintained is crucial for computing the probability of an accidental match, as each independent (x_M,y_M) term reduces the probability by more than an order of magnitude. Simply stated, poor ply integrity greatly decreases the probability of accidental match. However, because we seek an upper-bound to the probability of an accidental match, we are obliged to assume strict ply integrity unless we know otherwise.

The cutting and stitching errors are difficult to measure non-invasively in a real-world environment, thus an empirical characterization is impossible. The garment specifications limit the range of allowed cutting and stitching errors to about 1/8″ each, and it is reasonable to assume that these error distributions are normal. In the development below, we combine the cutting and stitching errors into a single Gaussian random variable with the standard deviation supplied by the user. Forensic examiners would typically use a conservative, small value, unless they had specific evidence otherwise. In all cases, the cutting and stitching error distribution is narrow compared to the distributions above, and has little influence on the incidence rate.

Marker Statistics

The position of each piece within the marker $(x_{M,i}, y_{M,i})$, is another potential source of variation, but only in some cases. If the markers used to make the two garments are independent, then the difference between the corresponding $(x_{M,i}, y_{M,i})$ coordinates are uniformly distributed on $[0..L) \times [0..L)$, independent for each piece. However, if the two garments are made from the same marker, then the $(x_{M,i}, y_{M,i})$ coordinates are the same and will cancel in Eq. (2). In this case, if any one-piece matches between the two garments, and ply integrity was maintained for both garments, then it is likely that all other pieces will also match.

Between the boundary cases of completely independent markers and identical markers is the case of similar markers, in which some of the pieces have the same relative positions. An example of two markers with a high degree of similarity (as observed by the authors) is shown in Fig. 6. These markers were created by the same designer, with just slightly different input data. But even here only six pieces out of 26 have similar positions in the layout, whereas most of the pieces are in different positions. Our research has revealed that markers with a few large pieces in the same position occur relatively often. This is due to two factors. First, all marker designs aim to minimize the amount of scrap material, and based on this both manual and automated marker designs employ a methodology of positioning the largest pieces first. Second, certain combinations of pieces fit well together. For example, the "back" and "side-back" pieces in the lower left corner of the markers in Fig. 6 fit tightly and leave little scrap.

The statistics of similar markers are difficult to characterize, and such a characterization would require access to a large number of sample markers, which manufacturers are reluctant to release for competitive reasons. To maintain a conservative upper-bound on the incidence rate, we must treat similar markers as identical.

The probability that two garments were made from the same marker is difficult to estimate. ACU manufacturers continuously discard and redesign their markers, while reusing some markers many times. Hence some markers may have been used for thousands of garments whereas others were used for only a few dozen. The manufacturer records describing the use of markers are not always kept and cannot be assumed to be accessible or exist. Nonetheless, if the probability that the two garments were made from

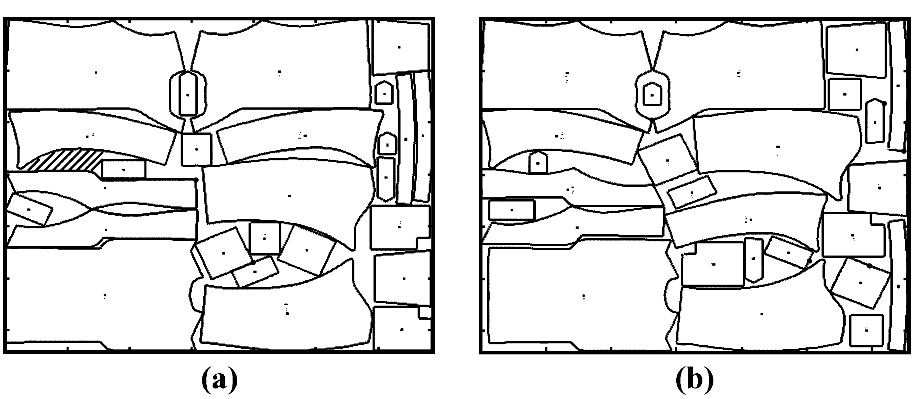


FIG. 6—*Similar single-garment markers for ACU coat: medium–long (a) and medium–short (b) sizes. Only six pieces have similar coordinates. Hashed area in (a) shows an example of a scrap piece.*

the same marker was known to be P_1, then it may be incorporated in the incidence rate calculation.

$$\Pr(\text{match}) = \Pr(\text{match}|\text{identical markers})P_1 + \Pr(\text{match}|\text{independent markers})(1 - P_1) \quad (7)$$

In some cases, it is impossible to estimate P_1, and the forensic examiner must assume the worst case, $P_1 = 1$. Overall, our research indicates that it is unlikely that two markers designed by different manufacturers are similar. This is because different manufacturers use differing spread lengths and fabric widths. Many manufacturers design large markers which contain the pieces for multiple garments. In this case, the number of possible arrangements becomes so large that the probability of similar markers becomes negligible, even for only a few pieces. A reasonable rule of thumb for upper-bounding the incidence rate is to assume identical markers for all garments made by the same manufacturer and independent markers for garments made by different manufacturers.

Probability Calculation

Suppose we have identified an n-piece match between a garment depicted in a surveillance image (garment A) and a garment seized from a suspect (garment B). We seek an upper-bound on the probability of this match occurring accidentally. The two sets of evidence are used to estimate a few parameters for each piece $i = 1\ldots n$, including the uncertainty Δ_i, the multiplicity m_i. The overall match may be described as follows

$$\forall i = 1\ldots n \quad \exists j \in \{1\ldots m_i\}: \quad \left|x_{i,A} - x_{i,B} - s_{x,i,j}\right| < \Delta_i, \quad \left|y_{i,A} - y_{i,B} - s_{y,i,j}\right| < \Delta_i \quad (8)$$

Note that the match is separated into longitudinal and transverse directions. This is tantamount to a square matching area, with sides of length $2\Delta_i$. Ideally, a circular matching area with radius Δ_i may be desirable; however, this would complicate the calculation significantly with little extra knowledge gained. As we are calculating an upper-bound to the match probability, the larger square matching area is consistent with the desired bound. The statistical characterization above revealed that the longitudinal and transverse coordinates can reasonably be modeled as independent; hence the probability calculation is separable

$$\Pr(\text{A and B match}) = \Pr(\text{A and B match longitudinally}) \cdot \Pr(\text{A and B match transversely}) \quad (9)$$

For the probability calculation, the coordinates above are broken into individual constituents, as in Eq. (3), and treated as random variables. The distributions of each random variable are summarized below. Define the following difference random variables

$$\begin{aligned} &\Delta x_M = x_{M,A} - x_{M,B}, \quad \Delta y_M = y_{M,A} - y_{M,B}, \\ &\Delta x_{M,i} = x_{M,i,A} - x_{M,i,B}, \quad \Delta y_M = y_{M,i,A} - y_{M,i,B}, \\ &\Delta x_{cs,i} = x_{cs,i,A} - x_{cs.i,B}, \quad \Delta y_{cs,i} = y_{cs,i,A} - y_{cs,i,B} \end{aligned} \quad (10)$$

Cutting and stitching errors are independent from each other, and independent for each piece in each garment. Accordingly, if each is modeled as normal, then their linear combination is also normal (17).

$$\Delta x_{cs,i} \sim N\left(0,\ \sigma_{cs}^2\right), \quad \Delta y_{cs,i} \sim N\left(0,\ \sigma_{cs}^2\right), \quad \sigma_{cs}^2 = \sigma_{cs,A}^2 + \sigma_{cs,B}^2 \quad (11)$$

Because the longitudinal coordinates are uniformly distributed in the $[0..L)$ periodic pattern, their difference Δx_M is also uniformly distributed over the same interval. The transverse coordinates are normally distributed; hence their difference is also normal, with doubled variance.

$$\Delta x_M \sim U(0,\ L), \quad \Delta y_M \sim N\left(0,\ 2\sigma_T^2\right) \quad (12)$$

The piece coordinate differences within each marker ($\Delta x_{M,i}$, $\Delta y_{M,i}$) are treated differently depending upon whether the markers are assumed to be independent or identical (similar markers are treated as identical so as to satisfy the upper-bound). For the independent marker case, these random variables are uniform on the interval $[0..L)$. For the identical marker case, these terms become zero. We describe the independent marker case next, followed by the identical marker case.

Independent Markers

If the markers used to make garments A and B are independent, then the piece coordinate differences ($\Delta x_{M,i}$, $\Delta y_{M,i}$) are also independent and uniformly distributed over the pattern period. In this case the ($\Delta x_{M,i}$, $\Delta y_{M,i}$) random variables subsume the other coordinate variables, and the overall coordinate differences are independent and uniform.

$$\begin{aligned} &\forall i = 1\ldots n\ \forall j = 1\ldots m_i: \\ &\Delta x_{i,AB} = x_{i,A} - x_{i,B} = \Delta x_M + \Delta x_{M,i} + \Delta x_{cs,i} \sim U(0,L), \\ &\Delta y_{i,AB} = y_{i,A} - y_{i,B} = \Delta y_M + \Delta y_{M,i} + \Delta y_{cs,i} \sim U(0,L) \end{aligned} \quad (13)$$

The probability of a match becomes the product of the probability for each piece individually, and can be separated into longitudinal and transverse components. Furthermore, the collection of m_i siblings where a match can occur are never close to each other (within 1–2″), hence for each piece the m_i potential matches with the pattern are mutually exclusive events, and their probabilities can simply be summed. In this case, the probability can be solved as

$$\begin{aligned} &\Pr(\text{match}\,|\,\text{independent markers}) \\ &= \Pr\left(\forall i = 1\ldots n\ \exists j \in \{1\ldots m_i\}: \left|\Delta x_{i,AB} - s_{x,i,j}\right| < \Delta_i,\ \left|\Delta y_{i,AB} - s_{y,i,j}\right| < \Delta_i\right) \\ &= \prod_{i=1..n} \sum_{j=1..m_i} \underbrace{\Pr\left(\left|\Delta x_{i,AB} - s_{x,i,j}\right| < \Delta_i\right)}_{2\Delta_i/L} \cdot \underbrace{\Pr\left(\left|\Delta y_{i,AB} - s_{y,i,j}\right| < \Delta_i\right)}_{2\Delta_i/L} \\ &= \prod_{i=1..n} \frac{4 m_i \Delta_i^2}{L^2} \end{aligned} \quad (14)$$

For typical values of the match uncertainty, Δ_i~0.5″and pattern period L~34″, the probability of match for each piece is on the order of *c.* 0.001. Hence, in the case of independent markers, the chance of a match for more than one to two pieces is extremely low. In contrast, match probabilities are not as low in the identical marker case, which is considered next.

Identical Markers

If garments A and B were manufactured from the same marker, then the piece coordinate differences ($\Delta x_{M,i}$, $\Delta y_{M,i}$) are zero. Furthermore, for all pieces that come from the same ply in the spread, the marker coordinate differences (Δx_M, Δy_M) are identical, and the overall coordinate differences for each piece (Δx_i, Δy_i) are highly correlated. In contrast, each piece for which ply integrity is not maintained will have an independent (Δx_M, Δy_M) term. Accordingly, the probability of the overall garment match can be separated into the probability that all pieces with ply integrity match and the probability that each piece without ply integrity matches.

$$\Pr(\text{all pieces match}) = \Pr(\text{all PI pieces match}) \cdot \prod_{i \in \text{NPI}} \Pr(i\text{-th piece matches}), \tag{15}$$

where PI denotes the subset of pieces that are manufactured with ply integrity and NPI denotes the subset without ply integrity.

Consider first the pieces without ply integrity. Each piece is independent of the others, and the longitudinal and transverse directions for each piece are separable. Furthermore, because Δx_M is uniformly distributed, the longitudinal component of the probability is independent of $s_{x,i,j}$. Using Bayes' Law, the match probability can be expressed at (17).

$$\begin{aligned}
&\Pr(i\text{-th NPI piece matches})\\
&= \sum_{j=1}^{m_i} \Pr\left(|\Delta x_M + \Delta x_{cs,i} - s_{x,i,j}| < \Delta_i\right) \cdot \Pr\left(|\Delta y_M + \Delta y_{cs,i} - s_{y,i,j}| < \Delta_i\right)\\
&= \sum_{j=1}^{m_i} \left(\int \Pr\left(|\Delta x_M + \Delta x_{cs,i} - s_{x,i,j}| < \Delta_i \middle| \Delta x_M\right) \Pr(\Delta x_M) d\Delta x_M \right.\\
&\quad \left. \cdot \int \Pr\left(|\Delta y_M + \Delta y_{cs,i} - s_{y,i,j}| < \Delta_i \middle| \Delta y_M\right) \Pr(\Delta y_M) d\Delta y_M \right)\\
&= \int \Pr\left(|\Delta x_M + \Delta x_{cs,i} - s_{x,i,j}| < \Delta_i \middle| \Delta x_M\right) \Pr(\Delta x_M) d\Delta x_M\\
&\quad \cdot \sum_{j=1}^{m_i} \int \Pr\left(|\Delta y_M + \Delta y_{cs,i} - s_{y,i,j}| < \Delta_i \middle| \Delta y_M\right) \Pr(\Delta y_M) d\Delta y_M
\end{aligned} \tag{16}$$

Evaluating the integral above is straightforward. $\Pr\left(|\Delta x_M + \Delta x_{cs,i} - s_{x,i,j}| < \Delta_i \middle| \Delta x_M\right)$ may be computed numerically by integrating over a portion of a normal curve (the PDF of Δx_{cs},i), or by using the standard normal cumulative distribution function (CDF).

$$\begin{aligned}
&\Pr\left(|\Delta x_M + \Delta x_{cs,i} - s_{x,i,j}| < \Delta_i \;\middle|\; \Delta x_M\right)\\
&= \Pr\left(-\Delta_i - \Delta x_M + s_{x,i,j} < \Delta x_{cs,i} < \Delta_i - \Delta x_M + s_{x,i,j} \;\middle|\; \Delta x_M\right)\\
&= \mathrm{CDF}_N\left(\frac{\Delta_i - \Delta x_M + s_{x,i,j}}{\sigma_{cs}}\right) - \mathrm{CDF}_N\left(\frac{-\Delta_i - \Delta x_M + s_{x,i,j}}{\sigma_{cs}}\right)
\end{aligned} \tag{17}$$

This applies to the transverse direction as well.

A simple upper-bound may be used to reduce the number of numerical integrations per NPI piece from m_i to 1. Consider the bottom integral over Δy_M in Eq. (16). The two Pr(…) terms under this integral are smooth, positive, symmetric functions of Δy_M with single maxima: at $\Delta y_M = s_{y,i,j}$ for $\Pr\left(|\Delta y_M + \Delta y_{cs,i} - s_{y,i,j}| < \Delta_i | \Delta y_M\right)$, and $\Delta y_M = 0$ for $\Pr(\Delta y_M)$. With this observation, it is straightforward to show that the integral over Δy_M has the highest value when $s_{y,i,j} = 0$ and decreases monotonically with increasing $|s_{y,i,j}|$ This implies that each spurious match (because of a sibling) contributes no more to the match probability that the true match at ($s_{x,i,j}$, $s_{y,i,j}$) = (0,0). In fact, with typical parameter values the probability becomes negligible if $s_{y,i,j}$ exceeds 1–2 inches. The probability of a match for the i-th NPI piece can therefore be upper-bounded by computing the probability assuming that $m_i = 1$ and then multiplying this probability by the number of siblings within a small transverse shift $|s_{y,i,j}| < 2(\Delta_i + \sigma_T + \sigma_{cs})$, which is defined as the *reduced multiplicity* of the match $m_{i,0} \le m_i$. Then

$$\Pr(i\text{-th NPI piece matches}) \le m_{i,0} \cdot \Pr(i\text{-th NPI piece matches} | m_i = 1) \tag{18}$$

Now consider the pieces with ply integrity. Their coordinate differences are dependent through the common marker coordinate differences (Δx_M, Δy_M), but they are conditionally independent given Δx_M and Δy_M. As such, the probability of all PI pieces matching can be expanded as follows.

$$\begin{aligned}
&\Pr(\text{all PI pieces match})\\
&= \iint \prod_{i \in PI} \Pr\left(\exists j \in \{1 \ldots m_i\} : \begin{array}{l} |\Delta x_M + \Delta x_{cs} - s_{x,i,j}| < \Delta_i \\ |\Delta y_M + \Delta y_{cs} - s_{y,i,j}| < \Delta_i \end{array} \middle| \Delta x_M, \Delta y_M \right)\\
&\quad \Pr(\Delta x_M) \Pr(\Delta y_M) d\Delta x_M d\Delta y_M\\
&= \iint \prod_{i \in PI} \left(\sum_{j=1}^{m_i} \Pr\left(\begin{array}{l} |\Delta x_M + \Delta x_{cs} - s_{x,i,j}| < \Delta_i \\ |\Delta y_M + \Delta y_{cs} - s_{y,i,j}| < \Delta_i \end{array} \middle| \Delta x_M, \Delta y_M \right) \right)\\
&\quad \Pr(\Delta x_M) \Pr(\Delta y_M) d\Delta x_M d\Delta y_M
\end{aligned} \tag{19}$$

Evaluating this integral is straightforward but potentially cumbersome. The summation over m_i in the bottom line of Eq. (19) can be replaced with a simple upper-bound using a similar argument to the NPI case, with one additional constraint. Because all pieces with ply integrity share the same marker shift (Δx_M, Δy_M), it is necessary that all PI pieces match *compatible* siblings simultaneously. The true match locations guarantee at least one set of compatible siblings, at $s_{(x,y),i,j=1} = (0, 0)$ for all i. Any other set of siblings must satisfy $\forall i, \exists j \in \{1..m_i\} : s_{x,i,j} \approx s_x$. Accordingly, let m_0 be the compatible multiplicity of the PI pieces:

$$\forall i \in \text{PI} \;\; \forall j \in (1..m_0) : \;\; |s_{x,i,j} - s_{x,j}| < 2(\Delta_i + \sigma_T + \sigma_{cs}) \;\cap\; |s_{y,i,j}| < 2(\Delta_i + \sigma_T + \sigma_{cs})$$

The probability that all PI pieces match is bounded by the product of the match probability assuming the multiplicity of each piece is one and the compatible multiplicity,

$$\Pr(\text{all PI pieces matches}) \le m_0 \cdot \Pr(\text{all PI pieces matches } | \forall i \in \text{PI} : m_i = 1) \tag{20}$$

Using the bound in Eq. (20), the longitudinal and transverse directions are again independent and the 2D integral in Eq. (19) can be replaced with the product of two 1D integrals. In practice, m_0 is virtually always one if multiple PI pieces are observed. In this case the probability calculation may be further simplified,

$$\begin{aligned}
&\Pr(\text{all PI pieces match})\\
&= \int \prod_{i \in PI} \Pr(|\Delta x_M + \Delta x_{cs}| < \Delta_i | \Delta x_M) \Pr(\Delta x_M) d\Delta x_M\\
&\quad \cdot \int \prod_{i \in PI} \Pr(|\Delta y_M + \Delta y_{cs}| < \Delta_i | \Delta y_M) \Pr(\Delta y_M) d\Delta y_M
\end{aligned} \tag{21}$$

Double-Sided Spreads

The calculation above assumes single-sided spreading. If double-sided spreading is used, an additional source of variation is introduced into the model and the probability of accidental match may be reduced. The mode of spreading affects the probability of accidental match only in the case that ply integrity is maintained and identical markers are used. In this section, we consider only PI pieces and identical markers. Furthermore, because double-sided spreading is only relevant in cases in which multiple PI pieces are observed, the probability that $m_0 > 1$ is negligible. The development below is valid for $m_0 = 1$. It is straightforward, although computationally expensive, to generalize the method for $m_0 > 1$.

When double-sided spreading is used, the pieces of each garment come from two adjacent plies in the spread, termed the *upper* and *lower* plies. Accordingly, there are now two pairs of marker shift differences instead of one pair: $(\Delta x_{M,u}, \Delta y_{M,u})$ and $(\Delta x_{M,l}, \Delta y_{M,l})$. These two pairs of differences are highly correlated, as described earlier. Conditioning the upper ply differences on the lower ply differences, it is clear that

$$\begin{aligned}\Pr\left(\Delta x_{M,u}\middle|\Delta x_{M,l}\right) &\sim N\left(\Delta x_{M,l},\ 2\sigma_{CL}^2\right)\\ \Pr\left(\Delta y_{M,u}\middle|\Delta y_{M,l}\right) &\sim N\left(r\Delta y_{M,l},\ 2\sigma_{CT}^2\right)\end{aligned} \tag{22}$$

Recall that $m_0 = 1$, hence the longitudinal and transverse directions can be separated. Considering the longitudinal component only, the probability can be expressed as

$$\begin{aligned}&\Pr(\text{all pieces match})\\ &= \int \Pr\left(\text{all pieces match } \middle|\Delta x_{M,l}\right)\Pr\left(\Delta x_{M,l}\right)d\Delta x_{M,l}\\ &= \int \Pr\left(\text{upper pieces match } \middle|\Delta x_{M,l}\right)\\ &\quad \Pr\left(\text{lower pieces match } \middle|\Delta x_{M,l}\right)\Pr\left(\Delta x_{M,l}\right)d\Delta x_{M,l}\end{aligned} \tag{23}$$

Once the upper and lower plies have been de-coupled, the match probability for the lower pieces is similar to the single-sided PI case presented earlier, and the match probability for the upper pieces may be expressed as

$$\begin{aligned}&\Pr\left(\text{upper pieces match}\middle|\Delta x_{M,l}\right)\\ &= \int \prod_{i\in\text{upperply}} \Pr\left(\left|\Delta x_{M,u}+\Delta x_{cs,i}\right|<\Delta_i\middle|\Delta x_{M,u},\ \Delta x_{M,l}\right)\\ &\quad \Pr\left(\Delta x_{M,u}\middle|\Delta x_{M,l}\right)d\Delta x_{M,u}\end{aligned} \tag{24}$$

In this case, a 2D integral is necessary, as the shift differences must be integrated over for both the upper and lower plies. Nonetheless, numerical evaluation of these integrals is straightforward.

Note that to evaluate Eq. (23), it is necessary to know whether each piece came from the upper or lower ply. Unless the specific marker used to manufacture the garment is available, it is typically impossible to know which ply each piece came from. If two mirror symmetric pieces are observed, such as left and right trouser legs, then it may be reasonable to assume that one comes from the upper ply and the other from the lower ply. For other pieces, the ply is modeled as an unknown variable, the probability calculated for every valid combination of upper/lower plies, and averaged. Of course, this iterative procedure may be quite slow if many pieces are observed. In the interest of speeding computation, and maintaining a valid upper-bound on the match probability, it may be preferable to simply assume that all pieces come from the same ply, and the probability reverts to the single-sided case.

Model Analysis

The development given in the previous two sections creates, perhaps, an overly complex picture of the statistical garment matching problem. In this section, we consider some numerical examples and heuristic rules that should clarify the statistical model.

Incidence Rate Trends

We first consider a few general examples with simplified parameters so as to highlight several important trends. Assume that the uncertainty values Δ_i are the same for all the matched pieces (in practice they are likely to be similar). Also, assume that the multiplicity for all pieces is 1, i.e., each piece is uniquely identified. In this case the upper-bound on the incidence rate depends only upon the number of observed pieces, their common match uncertainty Δ, and the manufacturing parameters. Figure 7 shows the upper-bound on the incidence rate as a function of the number of observed pieces for the case of identical markers, single-sided spreading, and typical manufacturer parameters. The incidence rate bound is shown for four levels of uncertainty, from $\Delta = 0.1''$ to $1''$.

In the case of identical markers, the main source of observable differences between garments is the longitudinal marker shift Δx_M, which is uniformly distributed over $[0..L)$. For $\Delta = 0.5''$, the chances of single longitudinal match is $2\Delta/L \sim 0.03$. When ply integrity is fully maintained (Fig. 7*a*), a longitudinal match for one piece implies likely matches for the other pieces as well (as they have common Δx_M). In this case, each additional observed piece reduces the incidence rate by only a small factor (~0.5–0.9) determined by the ratio of the uncertainty Δ to the cut/stitch standard deviation σ_{cs}. For large Δ (0.5–1.0″), the probability almost does not decrease with number of pieces. Each piece made without ply

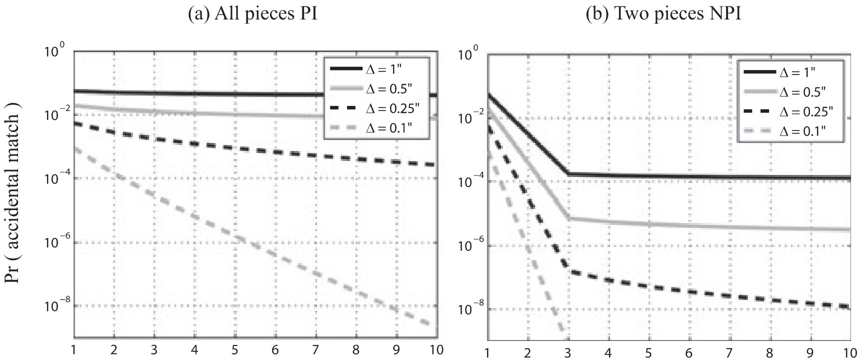


FIG. 7—*Upper-bound on the probability of accidental match as a function of number of matched pieces for different values of match uncertainty. Parameters: identical markers, single-sided spreading, $m_i = 1$, $\sigma_{cs} = 0.2''$, $\sigma_T = 0.35''$: (a) ply integrity is maintained for all pieces and (b) ply integrity is maintained for all but two pieces.*

integrity, however, has its own independent longitudinal shift Δx_M, and thus reduces the probability by a factor *c*. $2\Delta/L$. In Fig. 7*b*, just two NPI pieces reduce the incidence rate by orders of magnitude compared to Fig. 7*a*. Clearly, for the bounding the incidence rate for garments made with the same marker, ply integrity is a critical parameter that has to be accounted for.

When double-sided spreading is used, an additional source of observable differences arises because of marker shifts between upper and lower plies. Since these are not independent shifts the reduction in incidence rate is modest. This is illustrated in Fig. 8, which shows Pr(accidental match) computed with the same parameters as in Fig. 7*a*, but for double-sided spreading. The incidence rates are smaller by factors ~1.5–5 (higher values correspond to smaller match uncertainty Δ).

In the case of independent markers, both longitudinal and transverse shifts are uniformly distributed on [0..*L*) and hence the probability of an accidental match for a single piece is much lower than that for identical markers: $(2\Delta/L)^2 \sim 0.001$ (for $\Delta \sim 0.5''$). Moreover, the coordinate shifts are necessarily independent for each piece. Figure 9 shows Pr(match) for independent markers. As expected, the probabilities decrease more rapidly (with the number of matched pieces) relative to the identical markers case. Indeed, for three or more pieces, the incidence rate falls below the axis shown, indicating that the probability of such an accidental match is negligible with independent markers.

Piece multiplicity has the strongest effect in the case of independent markers, where it simply multiplies the probability of an accidental match. However, with the typical multiplicity values limited to $m_i \sim 1$–10, the probability of an accidental match is still very small. In the case of identical markers, the influence of piece multiplicity depends greatly on ply integrity. For pieces with ply integrity, piece multiplicity only influences the incidence rate if a common set of similar camouflage portions are located at the same shift for all pieces, which is unlikely in most cases. For pieces without ply integrity only those similar portions with small transverse shift yield a non-negligible probability, hence typically $m_{i,0} \sim 1$–4.

From these examples, a few loose rules-of-thumb emerge:

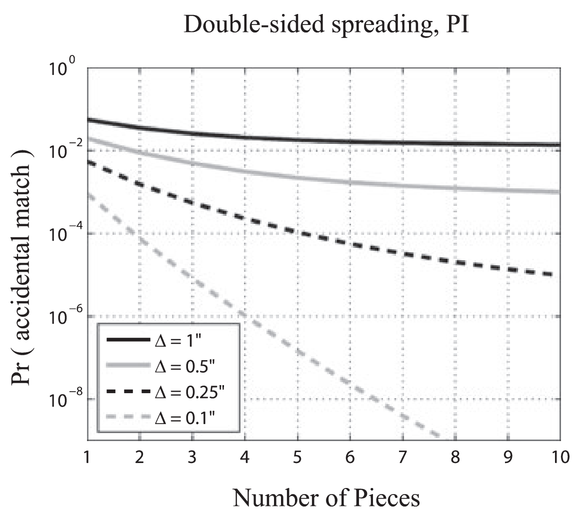


FIG. 8—*Probability of accidental match as a function of number of matched pieces for different values of matching uncertainty. Parameters: identical markers, double-sided spreading,* $m_i = 1$, $\sigma_{cs} = 0.2''$, $\sigma_T = 0.35''$, $\sigma_{CT} = 0.21''$, $\sigma_{CL} = 1.4''$, *ply integrity is maintained for all pieces.*

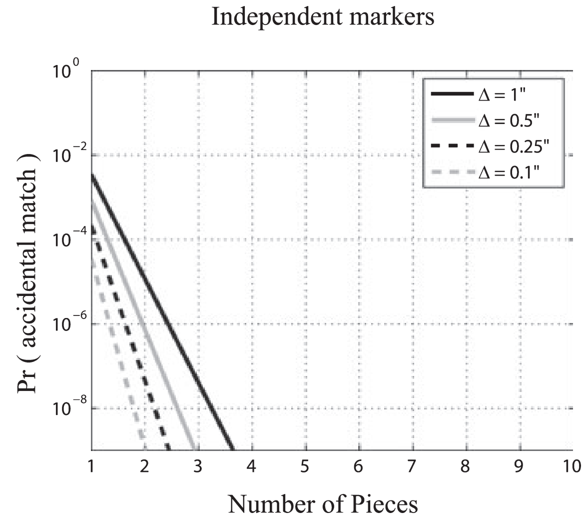


FIG. 9—*Probability of accidental match as a function of number of matched pieces for different values of matching uncertainty: independent markers.*

- For garments made from independent markers, each observable piece decreases the incidence rate by a factor of about m_i $(2\Delta/L)^2$.
- In the case of garments made from the same marker, there is strong dependence on ply integrity.

○ For pieces without ply integrity, each observable piece decreases the incidence by a factor of about $2m_{i,0}\Delta/L$.

○ For pieces with ply integrity, each observable piece contributes little to the overall uniqueness of the match, but the overall marker shift at least gives an incidence rate bound of about $2\Delta/L$.

Finally, it is worth noting that the incidence rate bound is not sensitive to the cut/stitch standard deviation, σ_{cs}. This particular manufacturing parameter is difficult to estimate, but contributes little to the statistical model in any event. The standard deviations of the other shift random variables are more significant, but the model is not especially sensitive to small perturbations in these parameters.

Application Example

As a practical example, consider a suspect wearing an ACU jacket, as depicted in Fig. 10. During the criminal investigation, a suspect is identified and a jacket that matches (Fig. 10) is seized. Four jacket pieces are qualitatively identified and matched (by an examiner) with the jacket seized from a suspect: the back, top sleeve right, collar, and elbow patch right. For the two larger pieces (back and elbow patch), the match uncertainty is estimated at 0.5″. For the two smaller pieces (collar and sleeve), the uncertainty is somewhat larger. In particular, the seams of these two pieces are not clearly observable; hence larger shifts in the pattern may go undetected. For these two pieces the match uncertainty is estimated at 0.7″.

Two of the pieces, the back and sleeve, are matched to unique portions of the camouflage pattern, hence their multiplicity is one. The other two pieces are found to match multiple portions of the camouflage pattern. The elbow patch, while relatively large, happens to be drawn from a portion of the pattern which is “echoed” elsewhere in the pattern, hence its multiplicity is two. The small

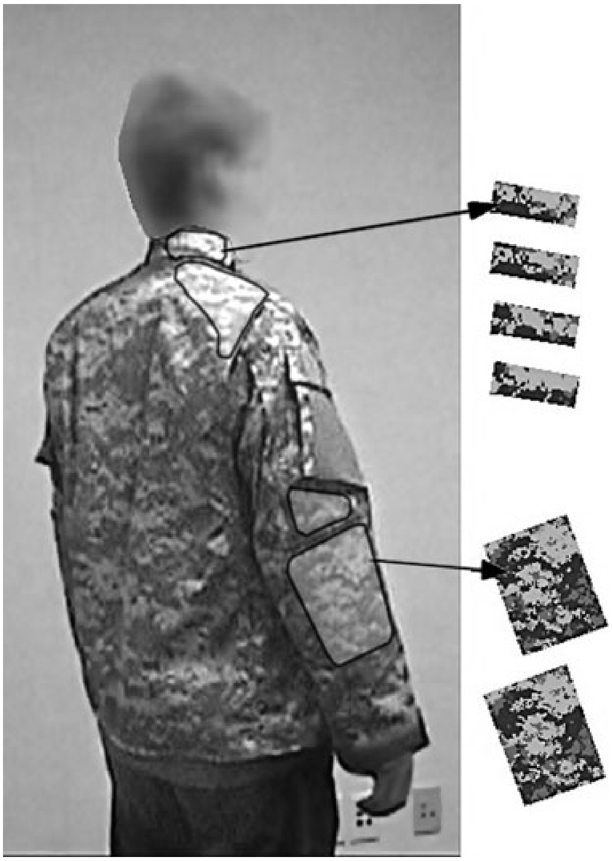


FIG. 10—*Example of statistical matching for an ACU jacket. Four pieces (back, right sleeve top, collar, and elbow patch) are identified and matched. For the back piece and sleeve piece, unique matches with the ACU pattern are established. The collar and elbow patch pieces are found to have four and two siblings, respectively.*

collar piece is found to have four siblings in the camouflage pattern.

Suppose the seized garment is found to have been manufactured at a known facility. All ACUs have labels indicating the manufacturer and contract number, which can be leveraged if available. This facility happens to use one-sided spreading and enforce strict ply integrity for all pieces, and $\sigma_{cs} = 0.141''$, $\sigma_T = 0.172''$. If the crime under investigation was committed on a military base, then a forensic examiner might conservatively assume that most of the jackets on this base were manufactured at the same facility as the jacket depicted in Fig. 10. For the purpose of bounding the incidence rate, a further conservative assumption is that all jackets on the base were manufactured using the same marker. In this case, the global multiplicity is one, $m_0 = 1$, and Eq. (20) is evaluated to bound the incidence rate at 2.40%.

On the other hand, if the manufacturing facility is known to not maintain ply integrity for all pieces, then the incidence rate falls significantly. Suppose ply integrity is only maintained for the back, sleeve and elbow patch, but not for the collar. In this case, the multiplicity of the collar is important. While a total of four siblings are found, only two are significant here. This is because only those siblings that are a small transverse shift from the true portion yield a non-negligible probability of match. Hence, for the collar piece $m_{i,0} = 2$. In this case, from Eq. (15), the incidence rate bound is 0.195%.

Finally, suppose that the jacket in Fig. 10 and the seized jacket are manufactured from independent markers. This would be the case, for example, if the military base purchased jackets from many different manufacturers and there was no evidence that the two jackets came from the same facility. In this case the incidence rate bound depends upon the full multiplicity of each piece, although the final incidence rate bound is still extremely low. Eq. (14) is evaluated to bound the incidence rate at 2.14×10^{-9}. This is, indeed, a strong piece of evidence.

Model Validation

To validate the model described above, an empirical study was conducted in which a large number of ACU jackets were sampled from one manufacturer. Empirical match probabilities were computed for identical marker case and compared to the probabilities predicted by the statistical model. This study both verified the accuracy of the upper-bound, as well as demonstrated that "typical-day" match probabilities are somewhat lower than the upper-bound.

Data Collection

A total of 639 complete ACU jackets were sampled as part of this study. Each jacket was sampled by placing it on an individual, who closed the zipper and Velcro front. The individual then stood 6 feet in front of a Hi-8 video camera (Sony Digital 8 Handycam) and two 7-megapixel cameras (Canon PowerShot A560) for ~20 sec with his arms held out at about 45°. One still was taken with each of the Canon cameras. The video camera ran continuously, and one frame was manually selected for each jacket. The two still images were "control" images, whereas the video image was further degraded and used as a "recovered" image. The video image was down-sampled by a factor of 1.5 and JPEG-compressed with a quality factor of 50, which resulted in images that were similar in quality to those of typical surveillance cameras. The large features in the camouflage pattern were still visible in the test images, but the fine details were lost and the edges and seams between pieces were indistinct. With this level of degradation, the average uncertainty was approximately $\Delta = 0.6''$. Portions of a sample control image and the corresponding recovered image are shown in Fig. 11.

The 639 jackets available at the manufacturer during the study were not all the same size; the largest group was 243 small-long jackets. Jackets of different size are made from different markers, and hence the incidence rate is necessarily very small. Therefore, the experiment below is constrained to only the 243 small-long jackets. This group of small-long jackets was manufactured with a single 6-jacket marker. Double-sided spreading was used, and the spreader contained two rolls of material that were rotated 180° relative to each other.

Worst-Case Groups

The focus of this study is the incidence rate of the "worst-case scenario" in which only the variables that are intrinsic to the manufacturing process are present. Of course, additional sources of variation are usually present, and these reduce the incidence rate. To estimate empirical match probabilities that correspond to the worst-case scenario, the 243 small-long jackets are divided into worst-case groups. These groups were determined by selecting a reference point for four pieces in each reference image: the left and right front pieces (considering only the shoulder portion of the front pieces), and the left and right chest pocket pieces. For example, the reference point for the shoulder piece shown in Fig. 11 is the top

FIG. 11—*A portion of a sample control image and the corresponding recovered image. The images used in the validation study were color.*

right corner of the Velcro patch. Using the surrounding pattern, the reference points were then located on the camouflage pattern using an automated pattern matching algorithm, and then manually inspected for errors and refined. A scatter plot of the reference points for the right front piece is shown in Fig. 12.

In a worst-case scenario, the reference points form a single "longitudinal line" due to the uniform longitudinal distribution and the tight normal transverse distribution. From Fig. 12, it is evident that the 243 right front pieces came from eight worst-case groups. From bottom to top, the numbers of pieces in the groups are (21, 34, 55, 20, 14, 34, 56, and 9). These groups are the result of the marker design and spreading method. The left front piece yielded the same eight worst-case groups, but for the left and right chest pockets the "longitudinal lines" overlapped and exhibited an additional source of variation, discussed below. Accordingly, eight worst-case groups were constructed based on the reference points of only the left and right front pieces.

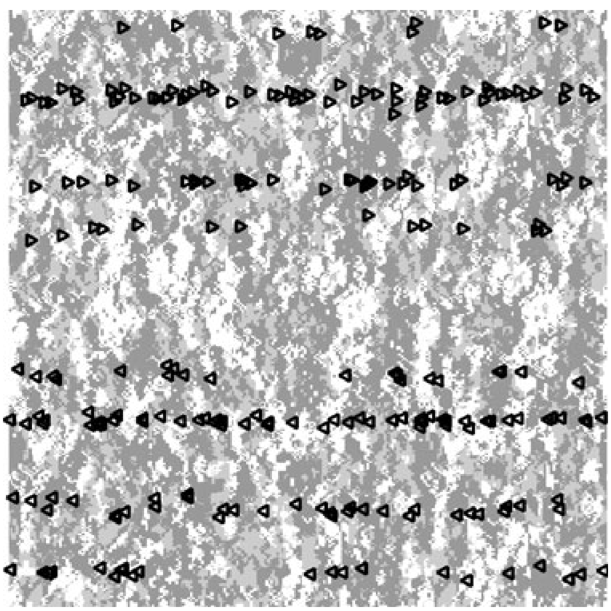


FIG. 12—*Scatter plot of reference points for the right front piece. The directions of the triangles indicate the relative 180° orientation of the piece.*

Matching Procedure

The six largest worst-case groups were included in the matching experiment. The two smallest groups were eliminated because of their statistical insignificance. For a group of size N, a total of $N(N-1)/2$ cross comparisons were performed for each piece. The total number of comparisons for each group was (210, 561, 1485, 190, 561, and 1540). The total number of comparisons for each piece was 4547.

The matching procedure was similar to other empirical studies of accidental match rates (9, 18). For jacket *A*, the control image and the recovered image are displayed on a computer screen, as shown in Fig. 11. The pair of images gives the human observer a clear example of the best possible match. The recovered image is then displayed for jacket *B*, and compared to the control image for jacket *A*. Four pieces are compared, the left and right front pieces and the left and right chest pockets. Note that for the front pieces, only the shoulder region (above the Velcro patch) is considered in the comparison, as this region is relatively free of wrinkles and shadows. For each piece, the human observer searched for significant differences that could not have been caused by wrinkles, shadows and other distortions because of repositioning. If no significant differences were observed, then a match was recorded.

One-Piece Incidence Rate

Empirical estimates of the incidence rate were computed from both the results of the manual matching experiment and from the reference points. The reference points empirical estimate was computed using the reference points and Eq. (2) with $\Delta = 0.6''$. For each piece, siblings were identified using the sibling finder tool in MUUSE (10). For the worst-case groups described above, there was close agreement between the matches found with the manual procedure and the matches found with the reference points. Both empirical estimates are reported below.

The statistical model was evaluated for the 243 left and right front pieces. About 85% of the pieces had multiplicity one ($m_{i,0} = 1$), and 15% of the pieces had multiplicity two ($m_{i,0} = 2$). For the pieces with multiplicity one, the incidence rate predicted by the model was 2.79%, and for the pieces with multiplicity two, the incidence rate was 5.58%. Table 1 shows the average predicted and empirical incidence rates, along with 95% confidence intervals for the empirical estimates. The empirical estimates were computed from the match results for the left and right front pieces, hence the

TABLE 1—*Worst-case one-piece match probabilities.*

	Model Prediction	Reference Point Estimate	Manual Match Estimate
Incidence rate (%)	3.21	3.02	3.18
95% Confidence interval		2.68–3.37	2.81–3.56

estimates are computed from a total of 9094 comparisons. Confidence intervals were computed using a bootstrap technique with 10,000 bootstrap samples (18). The incidence rates and confidence intervals were estimated individually for each worst-case group and then combined into a single estimate by weighting the estimates by the number of comparisons performed for each group. Both empirical estimates are in good agreement with the model prediction.

The table above shows worst-case incidence rates. It is also possible to estimate a "typical-day" incidence rate using the reference points for all 243 small-long jackets, without dividing the reference points into worst-case groups. In this case, a total of 29,403 comparisons were made for each of the four pieces. For the left and right front pieces, the average incidence rate was 0.521%, with a 95% confidence interval of 0.460–0.581%. For the left and right chest pocket pieces, the average incidence rate was 0.333%, with a 95% confidence interval of 0.283–0.383%. The incidence rate for the pocket pieces is lower because of an additional source of variation. The pocket pieces were rectangular, hence the shape of the pieces were invariant to 180° rotations. As the pocket pieces were sewn onto the front pieces, some pockets were rotated relative to the others, thus decreasing the match probability.

The typical-day probabilities are almost an order of magnitude lower than the worst-case probabilities. The disparity between the typical-day and worst-case probabilities can be reconciled if the specific marker and spreading procedure are known. Using the approximate ratios of the worst-case groups that result from the marker and spreading procedure, the typical-day probability can be computed from the worst-case probability.

Two-Piece Incidence Rate

Two-piece match probabilities were also estimated from both the reference points and the manual matching results. A preliminary analysis revealed that the most likely two-piece matches were matches of the front piece and the chest pocket attached to the front piece. The probabilities below consider just front/pocket matches.

Computing the incidence rate of multi-piece matches is complicated by the issue of ply integrity. Most ACU manufacturers claim to maintain strict ply integrity, and the underlying reason for ply integrity is to maintain color consistency. If the color consistency of the fabric is good, then occasional disruptions in ply integrity usually go unnoticed. Any departures from strict ply integrity introduce additional sources of variation that decrease the incidence rate significantly. The statistical model developed above assumes either strict ply integrity or none at all. In Table 2 both model predictions and both empirical estimates are reported. Using the same worst-case groups as in the one-piece case above, two empirical incidence rates for the front/pocket match are estimated. Both rates are estimated from a total of 9094 comparisons.

The two empirical estimates are in close agreement with each other, but not with either of the model predictions. The manufacturer claimed to maintain ply integrity, hence we would expect the empirical estimates to be close to the larger of the two model predictions. However, the model does not account for the additional 180° rotation variable for the pocket pieces, nor does the model account for loose ply integrity, which is common in practice. Unfortunately, both of these variables are difficult to characterize.

TABLE 2—*Worst-case two-piece match probabilities.*

	Model Prediction with Ply Integrity	Model Prediction no Ply Integrity	Reference Points Estimate	Manual Match Estimate
Probability (%)	2.41	0.111	0.561	0.495
95% Confidence interval			0.413–0.716	0.356–0.639

The reference points can also be used to estimate the typical-day incidence rate. In this case, a total of 58,806 front/pocket comparisons were made. The average incidence rate was 0.098%, with a 95% confidence interval of 0.073–0.124%. As for one-piece matches, the typical-day probability is significantly lower than the worst-case probability. The disparity between the two can be reconciled by accounting for the marker and spreading procedure, although the ambiguous ply integrity remains an issue.

Summary

The empirical results strongly support the statistical model. The empirical one-piece match probability for the worst-case scenario was in close agreement with probability predicted by the model. The overall empirical one-piece match probability was significantly lower than the worst-case probability because of the complex marker and spreading procedure. For two-piece matches, the comparison was complicated by the issue of ply integrity. The manufacturer appeared to maintain loose ply integrity, and we estimated a worst-case empirical probability between the predicted PI and NPI probabilities. As for one-piece matches, the overall empirical two-piece match probability was lower than the worst-case probability. In a practical case, if the forensic examiner can identify the serial number of the seized garment, then it may be possible to acquire the necessary information about the marker and spreading procedure to estimate a more realistic probability rather than the worst-case probability.

Discussion and Conclusion

The statistical evaluation of evidence is becoming more common in forensic investigations. Several recent studies demonstrate that no association can preclude the possibility of an accidental association with absolute certainty. In lieu of absolute certainty, more emphasis must be placed on quantifying, or bounding, the probability of an accidental association. In particular, it must be shown that the incidence rate is small in some specific sense.

A statistical model has been developed for upper-bounding the incidence rate of an association that has been established between a camouflage garment depicted in a surveillance image(s) and a garment seized from a suspect. This approach allows forensic examiners and prosecutors to determine the evidential value of the association.

The statistical model is possible because of the well-specified nature of manufacturing camouflage military garments and the enforcement of rigorous quality control. In this case, the dominant sources of variation can be characterized. For some sources of variation, a precise characterization is either unknown, or yields a statistical model that is mathematically unwieldy. Such random variables are either discarded or modeled with a tighter distribution so as to simplify computation and maintain an upper-bound on the overall probability of an accidental match. Any source of variation in the manufacturing process can only decrease the incidence rate. Hence ignoring a source of variation altogether never violates the upper-bound. All sources of variation that are intrinsic to the manufacturing process were accounted for so as to compute a worst-case match probability. Eight manufacturers were analyzed, and profiles computed for each.

An empirical study was conducted to validate the statistical model. By constructing worst-case groups, empirical match probabilities were estimated that were in close agreement with the predictions of the statistical model. Because the model only includes sources of variation that are common to all manufacturers, we would expect that the model can be similarly validated for any other manufacturer so long as its profile is known. The validation study also demonstrated that additional sources of variation are usually present, and that many of these sources of variation can be accounted for if the specific marker and spreading procedure are known.

The single most significant parameter in the statistical model is whether or not the two garments were manufactured with the same (or similar) markers. If the two garments can reasonably be assumed to come from independent markers, then the probability of an accidental match becomes negligible so long as three or more pieces are observed with reasonable precision in the surveillance image. However, there may be instances when this is an unreasonable assumption. For example, consider a crime that is committed on a military base where it is known that most uniforms were sourced from a single order to a single manufacturer. Hence it is more likely that two randomly chosen garments with the same size were made from the same marker. In this case the incidence rate can be as high as about 1 in 30 for U.S. ACU pattern.

If the two garments are assumed to come from the same marker, the question of ply integrity becomes important. Most manufacturers maintain ply integrity. In this case, somewhat counterintuitively, for typical low-quality surveillance images and manufacturing parameters, each additional matched piece does little to reduce the overall incidence rate. There is a single shift variable for the marker, but all observed pieces share the same marker shift. In contrast, if some pieces did not have ply integrity then these pieces may reduce the incidence rate significantly, even if small and poorly resolved in the surveillance image.

In general, garment association with surveillance images is difficult unless specific wear marks are visible or high-quality photographic images are available. Military camouflage garments present a unique context for introducing statistical methods into forensic investigations. In particular, an upper-bound on the incidence rate is calculated, which is compatible with current practices on evaluating statistical evidence in forensic science. This provides a natural extension to purely evidential or deterministic reasoning and enables a novel class of associative evidence to be considered in forensic investigations.

Acknowledgments

The authors wish to thank TSWG, including Jenna Farsetta and Shira Simon, for their assistance. The authors also wish to thank Dr. Chris Cole and Mr. William E. Pierce of Clemson Apparel Research, who served as garment design and manufacturing experts throughout the work. The authors wish to thank co-contributors at QS: Dr. Steve Rohde, Robert Lupa, John Lynch, Jeff Slutter, Phillip Munie, and Dr. William Williams. Finally, the authors wish to thank the manufacturers who cooperated in this study—without their help, this work may not have been possible.

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